Minimum of $f(x) = \frac{1}{x} + x$

Problem: Use AM-GM Inequality to show that Minimum of $f(x) = \frac{1}{x} + x$ is 2.

Solution:

The inequality is true for non-negative values of x. By using AM-GM inequality we have,

$$\frac{1}{2}\left(\frac{1}{x} + x\right) \geq \left(\frac{1}{x} \cdot x\right)^{\frac{1}{2}}$$

$$\frac{1}{x} + x \geq 2(1) = 2$$

$$\text{So, } f(x) \geq 2$$

This implies that for positive values of x, the minimum value of $f(x)$ is 2.

This occurs when $\frac{1}{x} = x$ or $x = 1$.

The graph below verifies that for positive values of x, the minimum point is $(1,2)$.